AMYGDALA

$z \mapsto z^2 + c$

A Newsletter of fractals & $\mathcal{M}$ (the Mandelbrot set)
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THE SLIDES (C18)

The four slides in this set were all created by Ken Philip.

457: Center = –1.71115.

535: SHVRB58.fs.
Center = –.7459,0601,58 + .0984,8530,361i, magnification = 10^9, escape radius = 2, dwell limit = 3825, program = MandelZot 1.3. A complex 'quad spiral' structure around a midget deep in Seahorse Valley. The previous batch had other views of this complex object. The effects of aliasing can produce surprising changes in the perceived patterns as the color mapping is altered, so it is worth experimenting with color mapping here.

543: *First Pinwheel.*
Center = –.1986,1627,510 + 1.1003,4606,584i, magnification = 5×10^8, escape radius = 2, dwell limit = 1000, program = MandelZot 1.3. A rerun in MandelZot of the first pinwheel I ever found, in a galaxy cluster on a tendril on the midget above the N wart. #543 uses a smooth CLUT, with different settings of the color animation cycle.

544: *Furry galaxies.*
Center = –1.7219,0586,9244,659 – .0022,0479,8027,793i, magnification = 10^{13}, escape radius = 2, dwell limit = 1000, program = MandelZot 1.3. A galaxy cluster off a midget on the spike, decorated with asymmetrically doubling spines.

SLIDE #547

Slide #547 was distributed with slide set C17, but only the name was given in Amy #17. Here is some more information about it.

547 (Ken Philip): *Furry galaxy pinwheel.1.* Center = –1.7219,0586,9244,66016 – .0022,0479,8027,79476i, magnification = 8.7×10^{14} escape radius = 2, dwell limit = 1000, program = MandelZot 1.3. A detail of the center of a galaxy cluster (#544) off a midget on the spike, decorated with asymmetrically doubling spines.

MORE SZILAGYI SLIDES

Steven Szilagyi has kindly supplied two more zoom sequences of 20 color slides each. These are available from Amygdala: see the Price List / Order Form for prices.

RENEWAL TIME!

Look at the label on the envelope this newsletter came in. In the lower right there is a box with one or two numbers in it, indicating when your newsletter subscription expires and when your slide subscription expires (if you get it). *N20,* for example, means that newsletter #20 is the last issue you will receive per your current subscription.

C16 means that slide set #16 was the last one you received. If that is what you've got, it means that as of this issue (#18) you've already missed two slide sets, #17 and #18! If you want to renew, here are the prices (postpaid) AS OF THIS DATE. Slide sets 17-22 (or any 6 sets, 4 slides each): \$19.20; or slide sets 17-26 (or any 10 sets, 4 slides each): \$32.00. If you've reading this much after September, 1989, please refer to a current price list.

FRACTALS AND HDTV

A short piece entitled *GEOMETRY COULD GIVE HDTV SIGNALS THE RIGHT SHAPE* appeared in Business Week for April 3, 1989, page 110:

The wide-screen, supercrisp pictures promised by high-definition TV require much more information than ordinary telecasts carry. Trying to squeeze an HDTV signal into existing broadcast channels, as the FCC has mandated, is giving many researchers grief. But mathematicians Michael F. Barnsley and Alan D. Sloan at Georgia Institute of Technology may have a solution: fractal geometry.

Fractals are those seemingly haphazard lines that turn out to have a strong kind of order. They are helping scientists gain insights into patterns hidden within meandering shapes such as coastlines and mountaintops. Barnsley and Sloan say the esoteric formulas can be used to encode an image — and squeeze it to a mere 1/1000 of its original size.

They believe the approach offers a way to compress a TV signal for transmission, then reverse the process in the TV set. Barnsley hopes to develop a system by year end. Meanwhile, the duo's Atlanta-based startup, Iterated Systems Inc., in February started marketing a $32,500 software package for creating models of biological systems from fractals.

— Sent in by Richard Morley

High-definition pictures without HDTV signals?, from *Machine Design*(May 25, 1989) p 15.

An Atlanta-based company called Iterated Systems Inc. hopes to generate high-definition television (HDTV) pictures from an ordinary broadcast signal. The trick is an automatic enhancement system which generates high-resolution images in real time. ISI's system will use a black box installed on HDTV receivers. Since the new technology is only on the receiving end, the usual problems associated with HDTV broadcasting are avoided. Says ISI President Alan Sloan, "We require no changes to the transmitted signal, so our technique is compatible with current NTSC broadcasting. Moreover, our system can be used with any of the HDTVs currently being proposed, so we are not worried about the adoption of standards."

Iterated Systems' technique is based on fractal geometry, which converts the incoming signal to a fractal representation in the form of a digital code. The fractal representation is independent of the display device and so the image can be viewed as video having higher resolution. Says Michael Barnsley, executive vice-president of Iterated Systems, "Because the fractal representation has no scale attached to it, you can recreate the image to whatever resolution you want."

A fractal is basically a complex geometric shape with very low information content. "In other words," says Barnsley, "fractals can produce visually complex and realistic images and yet can be specified and controlled with minimal amounts of data."

One result of Iterated Systems' work toward high-definition television is a Video Rate Decoder that is expected to debut this summer. Iterated Systems intends to store 45 seconds of video onto a high-density floppy disk that can, through the decoder, be played back on a PC/AT computer. Says Barnsley, "The image is high resolution and the quality is good. It's not HDTV but it does use a similar technology."

— Sent in by A. R. Wadum

DWELL FREQUENCY PEAKS IN MIDGETS ON THE SPIKE

— Ken Philip

In the description of color slide #144 (by Andrew LaMance) in AMYGDALA #13, there was a diagram showing peaks (for every fourth dwell value) in a dwell frequency plot for the midget at -1.94254 on the spike (See Figure 1). This behavior struck me as interesting, and I decided to look into it further. [Note: The center of the midget itself is approximately at -1.942525. -1.94254 refers to a point a bit to the left of the center of the head of the midget.]

The first test I made, after modifying a B&W True BASIC Mandelbrot program to produce dwell frequency histograms, was to verify LaMance's plot for this midget when the escape radius was set at 2, and then replot the histogram with

LaMance's Midget

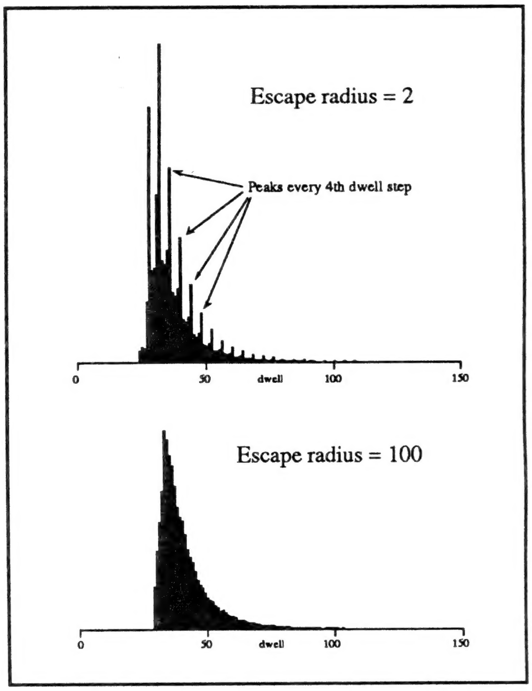

Figure 1 — Dwell Frequency Plot

Re=-1.94254, Im=0, mag=13221, max dwell=256 an escape radius of 100. I suspected that the peaks might be an artifact of using 2 as the escape radius, and thus was not surprised to see no trace of the peaks on the ER=100 plot (see Figure 1 on the previous page for a comparison of the two histograms). I then ran two B&W prints of the midget with escape radii of 2 and 100 (Fig. 2) to see what on the images corresponded to the dwell peaks. It was clear (Fig. 2a) that the bands in the ER=2 image with the very wide lobes were responsible for the dwell peaks in the histogram.

LaMance's Midget

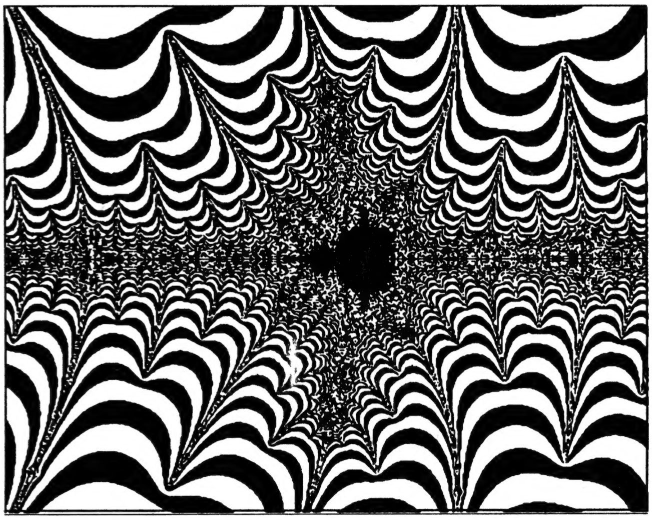

Figure 2
Re=-1.94254, Im=0, mag=13221, dwell=256
Escape radius=100

LaMance's Midget

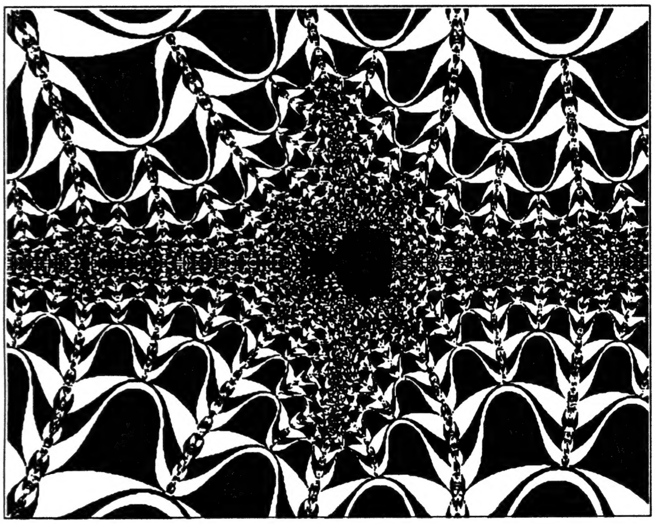

Figure 2a
Re=-1.94254, Im=0, mag=13221, dwell=256
Escape radius=2

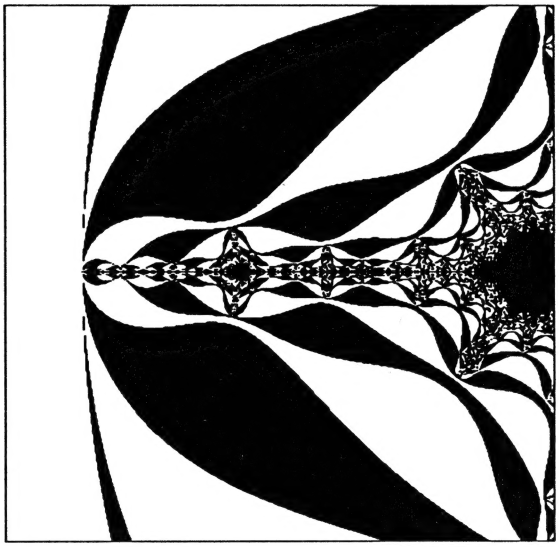

Figure 3
Spike: showing dwell bands

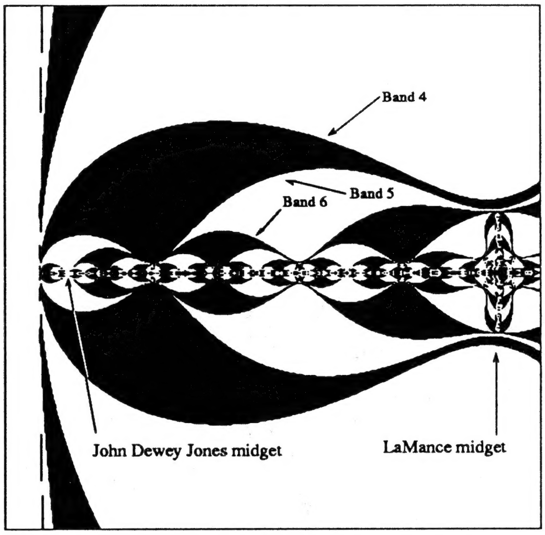

Figure 4
Spike tip: showing dwell bands

Figures 3 and 4 (showing the entire spike, and a magnified view of the end of the spike) indicate that the LaMance midget is formed by a compression of the 4th dwell contour (in an ER=2 plot). It occurred to me to wonder if other midgets would manifest the same behavior, and if there was any correlation between the number of the outermost dwell contour that was compressed around the midget, and the spacing of the dwell frequency peaks. Figure 5 (next page) shows the dwell frequency plot (ER=2 and ER=100) for the John Dewey Jones midget, formed by a compression in the 6th dwell contour, and Figure 6 (next page) shows the actual midget with ER=2 (Fig. 6a) and ER=100 (Fig. 6b). It is clear that the dwell peaks occur for ER=2, but at every 6th dwell

John Dewey Jones' Midget

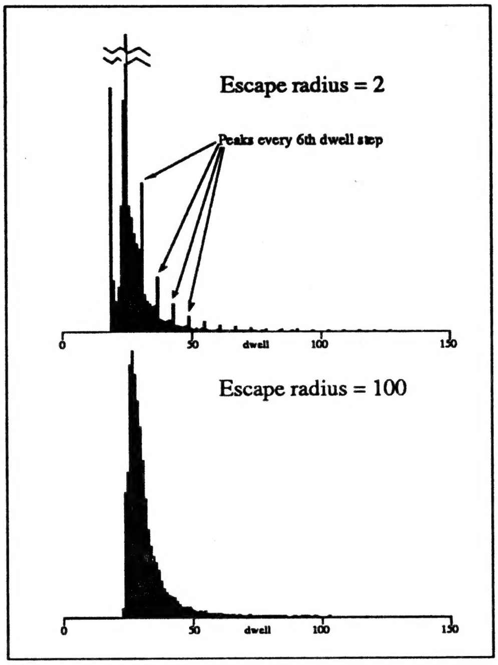

Figure 5
Dwell Frequency Plot
Re=-1.99638, Im=0, mag=150000, max dwell=256

John Dewey Jones' Midget

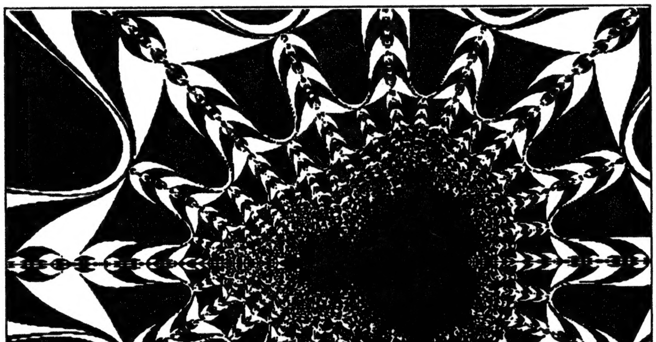

Figure 6a
Re=-1.99638, Im=0, mag=150000, dwell=256
Escape radius=2

value. Again, for ER=100 there are no such peaks.

I ran dwell frequency plots (ER=2) on the Biggest Midget, which is formed by a compression in the 3rd dwell contour—and on a midget at -1.861534 which is formed by a compression in the 5th dwell contour. Figure 7 shows the results: the peaks are not so pronounced as with the two preceding midgets, but they are present, and at a spacing which correlates with the outermost dwell band compression.

John Dewey Jones' Midget

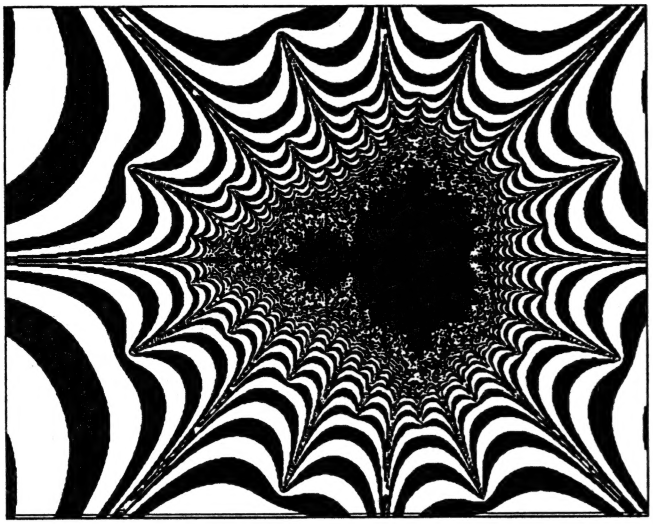

Figure 6b
Re=-1.99638, Im=0, mag=150000, dwell=256
Escape radius=100

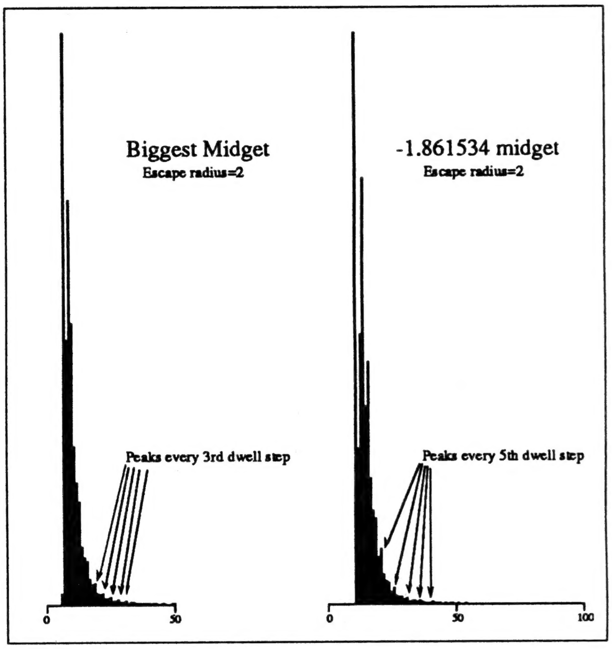

Figure 7
Dwell Frequency Plots

Figure 8 (next page) shows the more pronounced peaks on the midget at -1.985444, with a peak at every 5th dwell contour. This midget is also formed by a compression of the 5th dwell band.

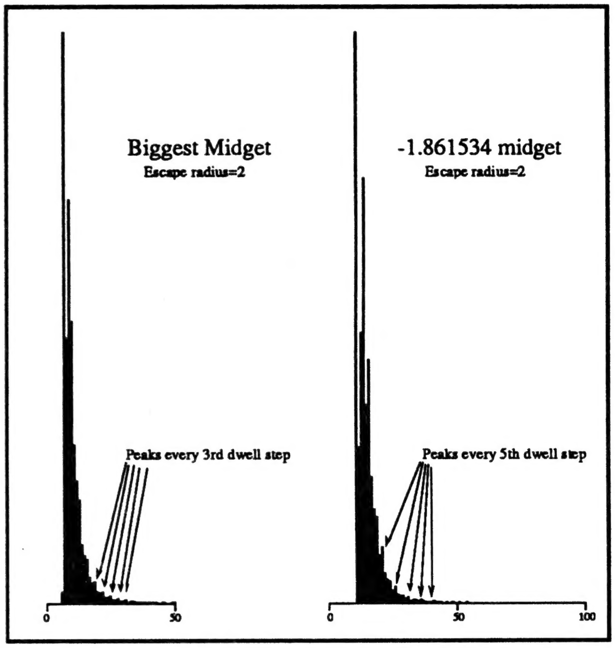

Figure 7
Dwell Frequency Plots

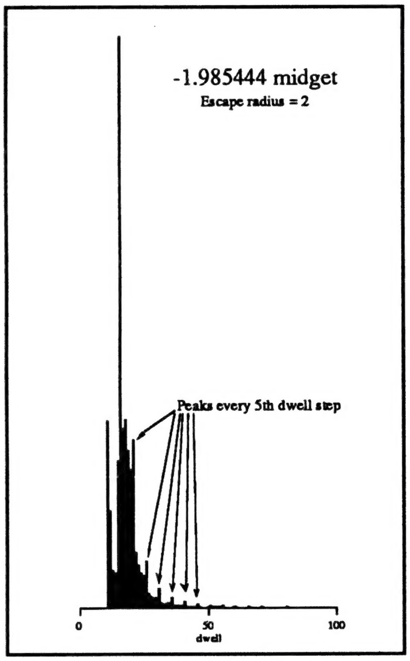

Figure 8a
Dwell Frequency Plot

The cycle number of the Biggest Midget is 3, and a run with a modified version of my WormTracks program indicated that the John Dewey Jones midget may have a cycle number of 6. Before one jumps to the conclusion that the cycle number is always equal to the dwell number of the outermost band compression, and in turn controls the dwell frequency peaks, I should mention that a WormTracks run showed 5, rather than 4, fixed points for the LaMance midget (and one of these was rather wide, suggesting that under higher magnification it might be double, which would give a total of 6 fixed points). So the relationship between the cycle number and the dwell peaks does not seem quite clear yet, although there may be such a correlation for midgets at or near the ends of their 'series' (as: the JDJ midget is the last midget formed by a compression of the 6th band).

I leave it to the mathematicians to explain *why* an escape radius of 2 gives rise to this phenomenon... It is clear, however, that using an escape radius of 2 means that you are restricting the entire 'universe' to the inside of a circle about the origin with that radius — since you can obtain no data from beyond the escape radius with a conventional Mandelbrot program. On a purely intuitive basis, using an escape radius large compared to the size of the Mandelbrot Set would appear more useful, since that allows you to follow dwell contours out to large distances.

-1.985444 midget

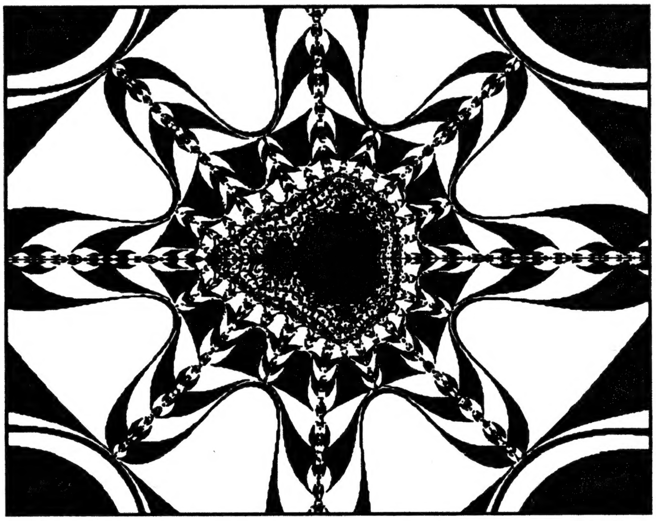

Figure 8b
Re=-1.985444, Im=0, mag=6000, dwell=200
Escape radius=2

TUTORIAL — THE BINOMIAL EQUATION (PART 2)

—*RS*

(Continued from Amy 16)

In this installment we are going to give solutions to the exercises posed at the end of the last installment.

1. *Express $\cos 4\theta$ and $\cos 5\theta$ in terms of $\cos\theta$ and $\sin\theta$.*

De Moivre's formula can be used to express $\cos 4\theta$ in terms of $\cos\theta$ and $\sin\theta$. Taking $n = 4$, (16.3) becomes:

$$\cos 4\theta + i \sin 4\theta = (\cos\theta + i \sin\theta)^4 \qquad (18.1)$$

The binomial formula (16.4) applied to the case $n = 4$ gives:

$$(a+b)^4 = \sum_{m=0}^{4} \binom{4}{m} a^m b^{4-m} \qquad (18.2)$$

The coefficients $\binom{4}{0}, \dots, \binom{4}{4}$ are 1, 4, 6, 4, 1, so (18.2) becomes:

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \qquad (18.3)$$

Using (18.3) to expand the right hand side of (18.1) we get:

$$\begin{aligned}\cos 4\theta + i \sin 4\theta = {}\\ \cos^4\theta + 4i\cos^3\theta \sin\theta - 6\cos^2\theta\sin^2\theta\\ - 4i\cos\theta\sin^3\theta + \sin^4\theta \qquad (18.4)\end{aligned}$$

If we equate the real parts of the two sides of (18.4) separately, we get:

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta.$$

The case $n = 5$ can be treated similarly. The binomial coefficients are 1, 5, 10, 10, 5, 1, and:

$$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$

The following identity will prove useful in solving problems 2 - 5:

$$(1 + z + \dots z^n)(1 - z) = 1 - z^{n+1} \qquad (18.5)$$

2. *Simplify $1 + \cos\theta + \cos 2\theta + \dots + \cos n\theta$ and $\sin\theta + \sin 2\theta + \dots + \sin n\theta$.*

If θ is a multiple of 2π, we have $1 + \cos\theta + \cos 2\theta + \dots + \cos n\theta = n+1$, and $\sin\theta + \sin 2\theta + \dots + \sin n\theta = 0$. Let us next suppose that θ isn't a multiple of 2π.

Using (16.3) for $n = 2, 3, \dots$, we get:

$$\begin{aligned}1 + (\cos\theta + i\sin\theta) + (\cos\theta + i\sin\theta)^2 + \dots +\\ (\cos\theta + i\sin\theta)^n = {}\\ 1 + \cos\theta + \cos 2\theta + \dots + \cos n\theta +\\ i(\sin\theta + \sin 2\theta + \dots + \sin n\theta) \qquad (18.6)\end{aligned}$$

Let $z = \cos\theta + i\sin\theta$, and note that $z \neq 1$, since θ is not a multiple of 2π. Thus from (18.5),

$$(1 + z + \dots + z^n) = (1 - z^{n+1})/(1 - z)$$

or

$$\begin{aligned}1 + (\cos\theta + i\sin\theta) + \dots + (\cos\theta + i\sin\theta)^n = {}\\ (1 - (\cos\theta + i\sin\theta)^{n+1})/(1 - \cos\theta - i\sin\theta)\\ (18.7)\end{aligned}$$

(18.6) and (18.7) yield:

$$\begin{aligned}1 + \cos\theta + \cos 2\theta + \dots + \cos n\theta +\\ i(\sin\theta + \sin 2\theta + \dots + \sin n\theta) = {}\\ (1 - (\cos\theta + i\sin\theta)^{n+1})/(1 - \cos\theta - i\sin\theta) \qquad (18.8)\end{aligned}$$

Equating real and imaginary parts of (18.8):

$$\begin{aligned}1 + \cos\theta + \cos 2\theta + \dots + \cos n\theta = {}\\ \Re[(1 - (\cos\theta + i\sin\theta)^{n+1})/(1 - \cos\theta - i\sin\theta)];\\ \sin\theta + \sin 2\theta + \dots + \sin n\theta = {}\\ \Im[(1 - (\cos\theta + i\sin\theta)^{n+1})/(1 - \cos\theta - i\sin\theta)].\end{aligned}$$

Does that qualify as a "simplification"?

3. *Express the fifth and tenth roots of unity in algebraic form.*

Since 1 is itself "a" fifth or tenth root of unity, I assume we're being asked to find *primitive* roots of unity.

Fifth root: $\omega_5 = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$.

Using (16.3) and letting $c = \cos\frac{2\pi}{5}$, $s = \sin\frac{2\pi}{5}$:

$$\begin{aligned}1 = {\omega_5}^5 = (c + is)^5 = {}\\ c^5 + 5ic4s - 10c^3s^2 - 10ic^2s^3 + 5cs^4 + is^5 \qquad (18.9)\end{aligned}$$

so $0 = 5c^4s - 10c^2s^3 + s^5$. We can divide by s, since $s \neq 0$: $0 = 5c^4 - 10c^2s^2 + s^4$. But $s^2 + c^2 = 1$, so $0 = 5(1-s^2)^2 - 10(1-s^2)s^2 + s^4 = 16s^4 - 20s^2 + 5$. The result is:

$$16s^4 - 20s^2 + 5 = 0 \qquad (18.10)$$

Whew! What started out looking like an intractable fifth degree equation, then a monstrous fourth degree equation, has turned into a pussycat quadratic in s^2. Solving (18.10) by the usual method, $s^2 = \frac{5 \pm \sqrt{5}}{8}$, or $s = \pm\sqrt{\frac{5 \pm \sqrt{5}}{8}}$. Which root?

Well, consulting a trig table or a calculator, $s = \sin\frac{2\pi}{5} =$ sin 72° is about 0.95, so we want the positive root: $s = \sqrt{\frac{5+\sqrt{5}}{8}}$. We then have $c = \sqrt{1 - s^2} = \pm\sqrt{1 - \frac{5+\sqrt{5}}{8}} = \pm\sqrt{\frac{3-\sqrt{5}}{8}}$, and since c = cos 72° ≈ 0.31, we take the positive root: $c = \sqrt{\frac{3-\sqrt{5}}{8}}$.

Finally, $\omega_5 = c + is$, which in algebraic terms is:

$$\omega_5 = \sqrt{\frac{3 - \sqrt{5}}{8}} + i\sqrt{\frac{5 + \sqrt{5}}{8}}.$$

Tenth root: As far as $\omega_{10} = \cos\frac{2\pi}{10} + i\sin\frac{2\pi}{10}$ goes, the same approach will not work, since the equation analogous to (18.9) is of too high degree to solve. Fortunately we have, by (16.3), ${\omega_{10}}^2 = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5} = \omega_5$, so we get :

$$\omega_{10} = \sqrt{\sqrt{\frac{3-\sqrt{5}}{8}} + i\sqrt{\frac{5+\sqrt{5}}{8}}}.$$

We need to tidy this up, i.e. to express ω_{10} in the form $a + ib$. If $\omega_{10} = a + ib$, then $(\omega_{10})^2 = (a + ib)^2 = a^2 - b^2 + 2iab$.

But ${\omega_{10}}^2 = \omega_5 = \sqrt{\frac{3 - \sqrt{5}}{8}} + i\sqrt{\frac{5 + \sqrt{5}}{8}}$, so we have:

$$a^2 - b^2 = \sqrt{\frac{3 - \sqrt{5}}{8}} = c, \qquad (18.11)$$

$$2ab = \sqrt{\frac{5 + \sqrt{5}}{8}} = s. \qquad (18.12)$$

(18.12) gives us $b = \frac{s}{2a}$, which substituted in (18.11) gives: $a^2-\left(\frac{s}{2a}\right)^2=c$. Therefore $a^4-ca^2-\frac{s^2}{4}=0$, whose solution is $a^2=\frac{c+1}{2}$, so $a=\sqrt{\frac{c+1}{2}}=\sqrt{\frac{1}{2}+\sqrt{\frac{3-\sqrt{5}}{32}}}$, and $b=\frac{s}{2a}=\sqrt{\frac{1}{2}-\sqrt{\frac{3-\sqrt{5}}{32}}}$. Summing this up:

$$\omega_{10}=\sqrt{\frac{1}{2}+\sqrt{\frac{3-\sqrt{5}}{32}}}+i\sqrt{\frac{1}{2}-\sqrt{\frac{3-\sqrt{5}}{32}}}.$$

4. *If* $\omega=\cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n}$, *prove that* $1+\omega^h+\omega^{2h}+\dots+\omega^{(n-1)h}=0$, *if h is an integer not a multiple of n.*

Let $z=\omega^h$. Note that: (1) $\omega^n=1$, and (2) since h is an integer not a multiple of n, $\cos\frac{2\pi h}{n}$ cannot be 1. Applying (16.3), $z=\left(\cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n}\right)^h=\cos\frac{2\pi h}{n}+i\sin\frac{2\pi h}{n}\neq 1$.

Therefore

$$1+\omega^h+\omega^{2h}+\dots+\omega^{(n-1)h}=$$
$$1+z+z^2+\dots+z^{(n-1)}=$$
$$\frac{z^n-1}{z-1}=\frac{\omega^{hn}-1}{z-1}=\frac{\omega^{nh}-1}{z-1}=\frac{1^h-1}{z-1}=0.$$

5. *What is the value of*
$S=1-\omega^h+\omega^{2h}-\dots+(-1)^{n-1}\omega^{(n-1)h}$?

Letting $z=-\omega^h$, the sum becomes

$$S=1+z+z^2+\dots+z^{n-1} \tag{18.13}$$

If $\omega^h=-1$, i.e. if $z=1$, then S is the sum of n ones, i.e.

$$\text{if } \omega^h=-1 \text{ then } S=n. \tag{18.14}$$

Let's deal with the other case: $\omega^h\neq-1$, i.e. $z\neq1$. Recalling that ω is an n-th root of unity, i.e. that $\omega^n=1$, we have $z^n=(-\omega^h)^n=(-1)^n(\omega^h)^n=(-1)^n$. Applying (18.5) to (18.13): $S=\frac{z^n-1}{z-1}=\frac{1-(-1)^n}{\omega^h+1}$, so if n is even, $S=0$, while if n is odd, $S=\frac{2}{\omega^h+1}$. Combining these results with (18.14) we can sum up:

If $\omega^h=-1$ then $S=n$;
if $\omega^h\neq-1$, then if n is even, $S=0$,
while if n is odd, $S=\frac{2}{\omega^h+1}$.

DIVCON ENHANCEMENTS

—Dave Platt

I've made a number of enhancements [in MandelZot] to the Divide and Conquer code. I first added some calls to the point-drawing code, so that the user can observe DivCon's progress around the perimeter of the rectangle under examination. Once I saw what DivCon was actually doing, I realized that a simple and effective optimization was possible, and incorporated the optimization into the code.

Briefly: before the code scans along one of the sides of a rectangle and computes the dwells, it performs a quick glance along the next "outermost" row or column. If all of the points immediately "outside" of the rectangle's side have already been calculated, and if their dwells match the dwell of the rectangle's corners, then the code skips the calculate-and-check pass on that side, and goes on to the next side. For example:

```
--------------------------------

----AAAAAAAAAAAAAAAAAAAAAAAA----

---CSSSSSSSSSSSSSSSSSSSSSSSSC---

--LS------------------------SR--

--LS------------------------SR--

--LS------------------------SR--

--LS------------------------SR--

---CSSSSSSSSSSSSSSSSSSSSSSSSC---

----BBBBBBBBBBBBBBBBBBBBBBBB----

--------------------------------
```

In this case, assume that the rectangle being scanned is denoted by the **S** (side) and **C** (corner) characters. The algorithm checks the four **C** corners to see if their dwells are the same and exits (or recurses) if not. If the **C**'s are the same, then the algorithm quickly scans the **A** (above) pixels; if their dwells have all been calculated and are the same as the **C** dwells, then the program assumes [provisionally] that all of the top-side **S** pixels would also have the same dwell, and doesn't bother to calculate them. If any **A** pixel hasn't been calculated, or doesn't match the **C** pixels, then the program runs along the top-side **S** pixels, calculating each one (if not already calculated) and comparing with **C**. An analogous check is made on the right (**R**), bottom (**B**), and left (**L**) sides.

This modification is a real win when the program is DivCon'ing an area that includes a large chunk of the M-set. If the rectangle being scanned lies entirely within M, then the program won't need to calculate the top-edge or left-edge pixels for this rectangle — the ones adjacent will already have been calculated in an earlier phase of DivCon. Net time savings for the rectangle in question: 25% to 50%.

In a "vanilla" DivCon run (in which the entire window is being scanned, and no points in the window have been scanned before), at most two sides of any rectangle-under-test will be adjacent to other already-scanned rectangles. In such runs, there's no advantage to performing the adjacent-row/column scan for the other two sides (bottom and right, in MandelZot's case). However, some programs may permit the user to select a portion of the window and run DivCon on only that portion. If the user does this several times to adjacent or overlapping portions of the window, then it's possible for a rectangle-under-examination to have three or (in degenerate cases) four sides adjacent to already-calculated portions of the window.

The scan of the row/column adjacent to the rectangle under consideration is very quick, because the program makes no attempt to calculate dwells for the points being scanned. This reduces the cost of a "failed" scan to a very small fraction of the benefit of a "succeeded" scan.

I've made one other small optimization to DivCon. Because the M-set and its surroundings are symmetrical with respect to the Y=0 axis, there's no need to run DivCon on pixel-rows where Y<0 if it has already been run for the corresponding Y≥0 rows. The top-level DivCon code in MandelZot now runs DivCon for the rows that cover Y≥0, and then copies the data to the corresponding rows for which Y<0. This speeds up the DivCon quite a bit when examining the whole M-set, or when looking at structures that straddle the axis of symmetry (e.g. the John Dewey Jones midget).

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126. CA Pickover, "Chaos and Halley's Method". IBM Report RC12747 [5/7/87] #57370. 10 pp. [Computational and graphic results on Halley's method for a one parameter family of rational functions. Striking full page plots of $x^7-1 = 0$ and $x^7+x^2-1 = 0$ at different magnifications. Lots of references.]

127. CA Pickover, "Overrelaxation and Chaos". Physics Letter A 130(3) [1988] 125-8. [More maths than in the above [#127] for Halley's method, but some very striking high quality images (2000x2000) in b/w. Details of how to produce them included.]

128. Prof. Haken (ed), "Springer Series in Synergetics" Springer-Verlag 1984. [In the period between 1977 and 1982 a number of international symposia on synergetics were held, most of them at Schloß Elmau in Bavaria. The meetings, under the direction of Professor Hermann Haken, brought together many leading scientists from rather diverse fields. The proceedings of these symposia were published by Springer. I [Reinald Eis] read a couple of these volumes and would like to recommend a few articles to the readers of Amygdala, because the topics dealt with pertinent fractals, too. I ought to include a cautionary word to readers who are easily frightened off by maths, even though most articles feature nice illustrations to improve digestibility.]

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